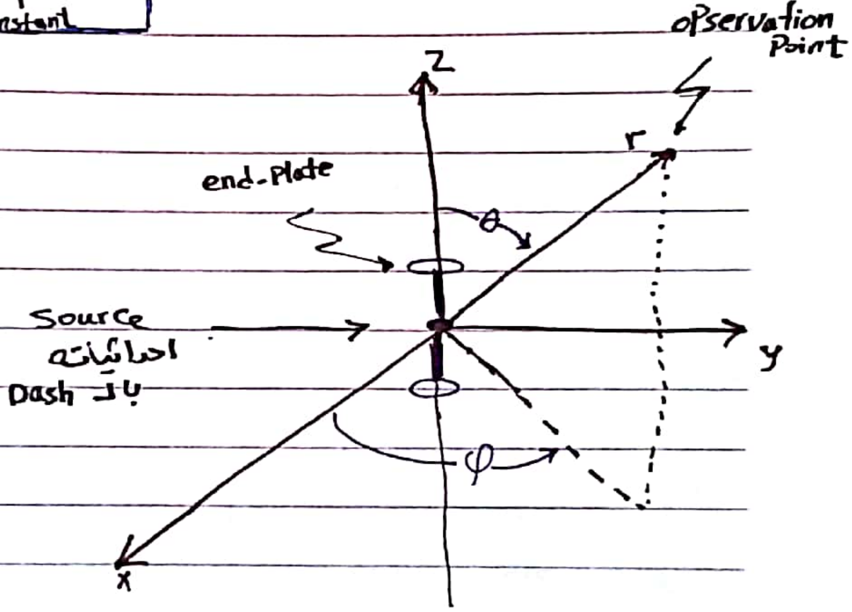
Chapter 4

Linear Wire Antenna

4.2) Infinitesimal dipole:

- → $L \ll \lambda$ or $L \leq \lambda/50$
- → Not Practical (used to represent a capacitor plate)
- → The variation of current is assumed to be constant & given by:

$$\boxed{I(z') = \underset{\uparrow \text{ constant}}{I_o}\, \hat{a}_z}$$



→ To find the field radiated by the current element, it will be required to determine First (A & F) ⇒ then find (E & H)

∴ The source only carries an electric current I_e

∴ I_m & the Potential function $\bar{F}$ are Zero

To Find $\bar{A}$

بنكامل على طول ال antenna

$$\bar{A}(x,y,z) = \frac{\mu}{4\pi} \int_c \bar{I}_e(x',y',z')\, \frac{e^{-jkR}}{R}\, dl'$$

احداثيات ال ~~Source~~ observation Point

احداثيات السورس Source

المسافة من اي نقطة على ال Source لل observation Point.

طول ال antenna $= dz'$

in our case:

$$\overline{I_e}(x',y',z') = I_o \hat{a}_z$$

$x' = y' = z' = 0$ (infinitesimal dipole)

$$R = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} = \sqrt{x^2+y^2+z^2} = r = \text{Constant.}$$

$$dl' = dz'$$

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\int_{-L/2}^{L/2} I_o \frac{e^{-jkr}}{r}\, dz' \; \vec{a_z}$$

$$= \frac{\mu I_o}{4\pi r} e^{-jkr} \int_{-L/2}^{L/2} dz' \; \vec{a_z}$$

$$= \frac{\mu I_o}{4\pi r} e^{-jkr} \Big[z' \Big]_{-L/2}^{L/2} \; \vec{a_z}$$

Cartesian Coordinate.

$$\boxed{\vec{A}(x,y,z) = \frac{\mu I_o L}{4\pi r} e^{-jkr} \; \vec{a_z}}$$

The next step is to find $\vec{H_A}$ using $\left(H_A = \frac{1}{\mu} \nabla \times \vec{A}\right)$

But first transform $\underline{A}$ from cartesian into spherical

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$A_x = 0$, $A_y = 0$

$$\boxed{\begin{aligned} \therefore A_r &= A_z \cos\theta = \frac{\mu I_o L}{4\pi r} e^{-jkr} \cos\theta \\ A_\theta &= -A_z \sin\theta = -\frac{\mu_o I_o L}{4\pi r} e^{-jkr} \sin\theta \\ A_\phi &= 0 \end{aligned}}$$

Remember:

$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \vec{a_u} & h_2 \vec{a_v} & h_3 \vec{a_w} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

where $h_1 = 1$, $h_2 = r$, $h_3 = r\sin\theta$

$$\therefore \vec{H_A} = \frac{1}{\mu} \nabla \times \vec{A} = \frac{1}{\mu}\left(\frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r a_\theta & r\sin\theta\, a_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & r A_\theta & r\sin\theta\, A_\phi \end{vmatrix} \right)$$

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بالأولى و بالتانية
بصفر

$$\vec{H} = \frac{1}{\mu}\left(\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & 0 \end{vmatrix}\right)$$

$$\vec{H} = \frac{1}{\mu}\left(\frac{1}{r^2\sin\theta}\left(r\sin\theta\,\vec{a_\phi}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right]\right)\right)$$

$$= \frac{1}{\mu r}\vec{a_\phi}\left[\frac{\partial}{\partial r}\left(-\frac{\mu I_o L}{4\pi}e^{-jkr}\sin\theta\right) - \frac{\partial}{\partial\theta}\left(\frac{\mu I_o L}{4\pi r}e^{-jkr}\cos\theta\right)\right]$$

$$\boxed{\vec{H} = \frac{jkIL}{4\pi r}e^{-jkr}\left[1+\frac{1}{jkr}\right]\sin\theta\,\vec{a_\phi}}$$

$\therefore$

$$\boxed{\begin{aligned} &H_r = H_\theta = 0 \\ &H_\phi = \frac{jkI_oL}{4\pi r}e^{-jkr}\left[1+\frac{1}{jkr}\right]\sin\theta \end{aligned}}$$

To find $\vec{E}$ $\Rightarrow$ $\vec{E} = \frac{1}{j\omega\epsilon}\nabla\times\vec{H}$

لكن ده هيبقى طويل جداً انى اعمل curl تانى

$\boxed{E_\theta = \eta H_\phi}$ $\Leftarrow$ بالسهل انى اقول

Summary

	infinitesimal dipole	far field ($kr \gg 1$)
	$E_r = \eta\frac{I_oL}{2\pi r^2}e^{-jkr}\left[1+\frac{1}{jkr}\right]\cos\theta$	$E_r = 0$
	$E_\theta = \eta\frac{jkI_oL}{4\pi r}e^{-jkr}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]\sin\theta$	$E_\theta = \eta\frac{jkI_oL}{4\pi r}e^{-jkr}\sin\theta$
	$E_\phi = 0$	$E_\phi = 0$
	$H_r = H_\theta = 0$	0
	$H_\phi = \frac{jkI_oL}{4\pi r}e^{-jkr}\left[1+\frac{1}{jkr}\right]\sin\theta$	$H_\phi = \frac{jkI_oL}{4\pi r}e^{-jkr}\sin\theta$

$$A \longrightarrow \underset{H}{E} \longrightarrow \vec{W} \xrightarrow{\int} P_{rad} \longrightarrow R_r$$

Power denisty

4.2.2 Power denisty:

$$\vec{W} = \frac{1}{2} \left[\vec{E} \times \vec{H}^* \right]$$

$$\vdots$$

$$\vec{W} = W_r \vec{a_r} + W_\theta \vec{a_\theta}$$

$$W_r = \frac{1}{2} E_\theta^* H_\varphi^* = \frac{\eta}{8} \left| \frac{I_o L}{\lambda} \right|^2 \frac{\sin^2\theta}{r^2} \left[1 - j \frac{1}{(kr)^3} \right]$$

Power radiated:

$$P = \oiint_s \vec{W}_{avg} \cdot \vec{ds} \leftarrow r^2 \sin\theta \, d\theta \, d\varphi \, \vec{a_r}$$

$$\vdots$$

at far field (radiated Power) (real) will be:

$$\boxed{P_{rad} = \eta \frac{\pi}{3} \left| \frac{I_o L}{\lambda} \right|^2} = \frac{1}{2} |I_o|^2 R_r$$

Radiated resistance:

$$R_r = 80 \pi^2 \left(\frac{L}{\lambda} \right)^2$$

$$L \leq \lambda/50$$

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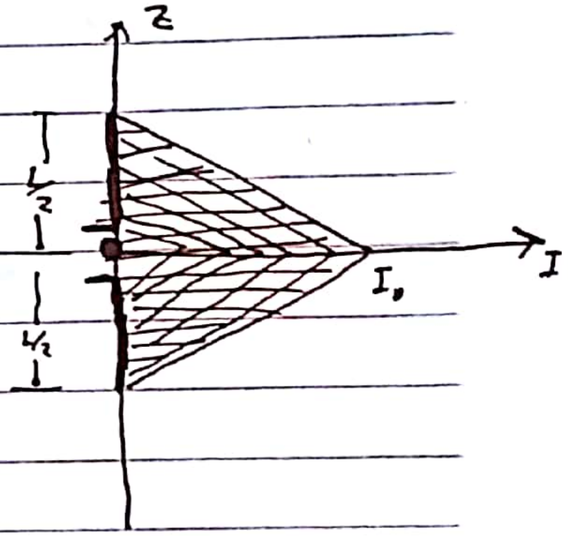
4.3) Small Dipole: $\lambda/50 < L \leq \lambda/10$

→ better approx. of current distribution of wire here is the triangular variation.

$$I_e(x',y',z') = I(z') = \begin{cases} I_0\left(1+\frac{2}{L}z'\right)\vec{a_z} & -\frac{L}{2} \leq z' \leq 0 \\ I_0\left(1-\frac{2}{L}z'\right)\vec{a_z} & 0 \leq z' \leq L/2 \end{cases}$$



→ ~~we can write $\vec{E}$~~ To find A

:

$$\vec{A}(x,y,z) = \frac{1}{2}\left[\frac{\mu I_0 l}{4\pi r}e^{-jkr}\right]\vec{a_z}$$

→ Thus we can write $\vec{E}$ & $\vec{H}$ fields radiated by a small Dipole antenna as:

infinitesimal Dipole (Far field)	small Dipole (Far field)
$E_\theta = \eta\frac{jkI_0L}{4\pi r}e^{-jkr}\sin\theta$	$E_\theta = \eta\frac{jkI_0L}{8\pi r}e^{-jkr}\sin\theta$
$H_\phi = \frac{jkI_0l}{4\pi r}e^{-jkr}\sin\theta$	$H_\phi = \frac{jkI_0L}{8\pi r}e^{-jkr}\sin\theta$
$E_r \cong E_\phi = H_r = H_\theta = 0$	. . . . = 0

Radiation Resistance:

$$R_r = 20\pi^2\left(\frac{L}{\lambda}\right)^2$$

4.5) Finite length Dipole:

→ For a very thin dipole antenna (ideally zero diameter), the current distr. can be written as.

$$I_e(x'=0, y'=0, z') = I(z') = \begin{cases} I_0 \sin\left[k\left(\frac{L}{2}+z'\right)\right] \vec{a_z} & -\frac{L}{2} \le z' \le 0 \\ I_0 \sin\left[k\left(\frac{L}{2}-z'\right)\right] \vec{a_z} & 0 < z' \le \frac{L}{2} \end{cases}$$

$$E_\theta = \eta \frac{j I_0}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin\theta}\right]$$

$$H_\phi = \frac{j I_0}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin\theta}\right]$$

$$\vec{W}_{avg} = \eta \frac{|I_0|^2}{8\pi^2 r^2} \left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\left(\frac{kL}{2}\right)}{\sin\theta}\right] \vec{a_r}$$

$$U = r^2 W_{avg}$$

4.6) Half wave Dipole

→ $L = \lambda/2$

→ $R_r = 73\,\Omega$ which is very $50\,\Omega$ ch's impedance of some T.L

→ The Electric & magnetic field can be obtained from finite length dipole field eq. by letting $L = \lambda/2$

$$E_\theta = \eta \frac{j I_0}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$ → approx $\sin^{2.5}\theta$

$$H_\phi = \frac{j I_0}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

avg Power density

$$\rightarrow \vec{W}_{avg} = \eta \frac{|I_0|^2}{8\pi^2 r^2} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \vec{a_r} \cong \eta \frac{|I_0|^2}{8\pi^2 r^2} \sin^3(\theta)\, \vec{a_r}$$

radiation intensity → $U = r^2 W_{avg} \cong \eta \frac{|I_0|^2}{8\pi^2} \sin^3(\theta)\, \vec{a_r}$

Questions

Remember:

infinitismal dipole:

Far field (Z-axis)	Generally
$E_\theta = \eta \frac{JKI_oL}{4\pi r} e^{-JKr} \sin\theta$	$E_\psi = \eta \frac{JKI_oL}{4\pi r} e^{-JKr} \sin\psi$
$H_\phi = \frac{JKI_oL}{4\pi r} e^{-JKr} \sin\theta = \frac{E_\theta}{\eta}$	$H_\chi = \frac{JKI_oL}{4\pi r} e^{-JKr} \sin\psi = \frac{E_\psi}{\eta}$
	where ψ: angle between the dipole axis & $\vec{a_r}$

Unit vector transformation:

	$\vec{a_x}$	$\vec{a_y}$	$\vec{a_z}$
$\vec{a_r}$	$\sin\theta\cos\phi$	$\sin\theta\sin\phi$	$\cos\theta$
$\vec{a_\theta}$	$\cos\theta\cos\phi$	$\cos\theta\sin\phi$	$-\sin\theta$
$\vec{a_\phi}$	$-\sin\phi$	$\cos\phi$	0

4.1) Given: horizontal infinitesimal dipole

$I = I_o$

Placed along x-axis

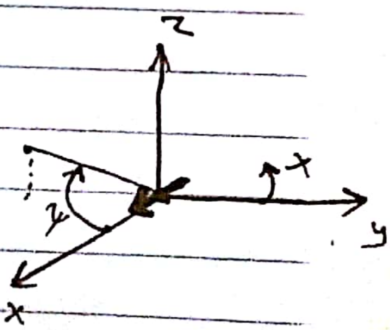
Required: Derive the directivity & far field radiation.

soln: $\because a_r \cdot a_{dipole} = |1|.|1|.\cos\psi$

$\therefore \sin\psi = \sqrt{1-\cos^2\psi}$

$= \sqrt{1-|\hat{a_x}\cdot\hat{a_r}|^2}$

$= \sqrt{1-(\sin\theta\cos\phi)^2}$



in far-zone fields

$E_\psi = \eta \frac{JKI_oL}{4\pi r} e^{-Jkr} \sin\psi = J\eta \frac{kI_oL}{4\pi r} e^{-Jkr} \sqrt{1-(\sin\theta\cos\phi)^2}$

$H_\chi = \frac{E_\psi}{\eta}$ #

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b) To find directivity:

$$\because U = U_0 \sin^2\psi$$
$$\therefore \quad = U_0 (1 - \sin^2\theta \cos^2\varphi$$

$$\because P_{rad} = \oiint_{\Omega} U \, d\Omega \leftarrow \sin\theta \, d\theta \, d\varphi$$

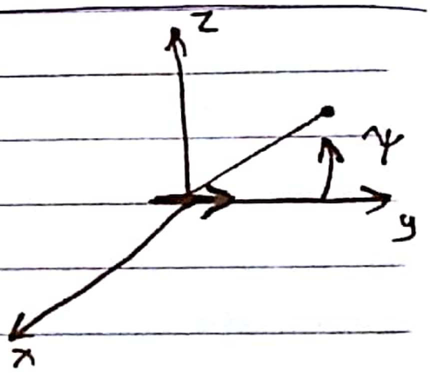
$$\because P_{rad} = U_0 \int_0^{2\pi}\int_0^{\pi} (1 - \sin^2\theta \cos^2\varphi) \cdot \sin\theta \, d\theta \, d\varphi$$
$$= U_0 \left[\int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\varphi - \int_0^{2\pi}\int_0^{\pi} \sin^3\theta \cos^2\varphi \, d\theta \, d\varphi \right.$$

Note $\int_0^{\pi} \sin^3\theta = 4/3$

$$\vdots$$
$$= U_0 \frac{8\pi}{3}$$

$$D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi U_0}{U_0 \, 8\pi/3} = 3/2 = 1.5 \quad \#$$

4.2) RePeat 4.1 along y-axis:



$$\because \sin\psi = \sqrt{1 - \cos^2\psi} = \sqrt{1 - |\hat{a}_y \cdot \hat{a}_r|^2}$$
$$= \sqrt{1 - \sin^2\theta \sin^2\varphi}$$

a) $E_\psi = j\eta \frac{kI_0L}{4\pi r} e^{-jkr} \sin\psi$

$$= j\eta \frac{kI_0L}{4\pi r} e^{-jkr} \sqrt{1 - \sin^2\theta \sin^2\varphi}$$

$H_x = \frac{E_\psi}{\eta}$

b) $U = U_0 \sin^2\psi = U_0 (1 - \sin^2\theta \sin^2\varphi)$

$$P_{rad} = \oiint_{\Omega} U \, d\Omega = U_0 \int_0^{2\pi}\int_0^{\pi} (1 - \sin^2\theta \sin^2\varphi) \sin\theta \, d\theta \, d\varphi$$
$$= U_0 \, 8\pi/3 \, U_0 \quad \#$$

$$D = \frac{4\pi U_{max}}{P_{rad}} = 3/2 = 1.5$$

4.30)

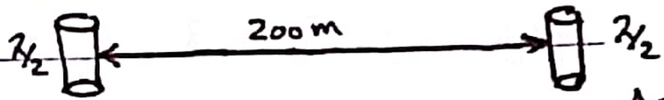
given: $\lambda/2$ dipole centered at origin $\Leftarrow$ Antenna 1 TX

$P_t = 600$ W

$f = 300$ MHZ

$\lambda/2$ dipole at $P(r, \theta, \phi)$ $r = 200$, $\theta = 90°$, $\phi = 40°$ $\Leftarrow$ RX

it is in parallel with the TX



$\theta = 90°$

$\phi = 40°$

Required: The available power at the second receiving dipole:

soln: At $f = 300$ MHZ

$$\lambda = c/f = 1\,\text{m}$$

$$\lambda/2 = 0.5\,\text{m}$$

From Friis transmission equation when reflection & polarization matches:

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{ot} G_{or}$$

G_{or} ← gain received

G_{ot} ← gain Transmitted

∵ For half wave dipole antenna D = max. gain = 1.64

assume lossless

$$\therefore G_{ot} = G_{or} = 1.64$$

$$\therefore P_r = \left(\frac{1}{4\pi \cdot 200}\right)^2 (1.64)(1.64)(600)$$

$$= 0.25\ \text{mW}$$ #

4.31) given : $\lambda/2$ dipole along z-axis

$P_{in} = 100W$

overall efficiency $= 50\%$

Required : Power denisty (w/m^2) at $r = 500m$ $\theta = 60°$ $\phi = 0°$

Soln :

$$\because W_{av} = \frac{1}{2}\frac{|E|^2}{\eta} = \eta \frac{|I_o|^2}{8\pi^2 r^2}\left[\frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}\right]$$

$$U = r^2 W_{avg} = \eta \frac{|I_o|^2}{8\pi^2}\left[\frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}\right]$$

$$P_{rad} = \oint_\Omega U\, d\Omega = \eta \frac{|I_o|^2}{8\pi^2}\int_0^{2\pi}\int_0^{\pi} \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}\sin\theta\, d\theta\, d\phi \quad (\sin^{2.5}\theta)^2$$

use approx $\cos(\pi/2 \cos\theta) = \sin^{2.5}\theta$

$$\therefore P_{rad} = \eta \frac{|I_o|^2}{4\pi}\int_0^{\pi} \sin^4\theta\, d\theta$$

$$= \eta \frac{|I_o|^2}{4\pi}\cdot\frac{3\pi}{8} = \eta \frac{3|I_o|^2}{32}$$

$\because P_{rad} = \frac{1}{2}|I_o|^2 R_r = \text{efficiency} * P_{in} = 0.5 * 100 = 50W$

$\because R_r = 73\Omega$

$$\therefore \frac{1}{2}|I_o|^2 \times 73 = 50 \Rightarrow \boxed{|I_o|^2 = 1.2}$$

$$W_{av} = 120\pi \cdot \frac{1.2}{8\pi^2(500)^2}\left[\frac{\cos^2(\pi/2 \cos 60)}{\sin^2 60°}\right] \quad 2/3$$

$$= 1.53 \times 10^{-5}\ W/m^2$$

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4.39) Given : $L = 3\text{cm}$

dipole

$I_o = 10 e^{j60}$ A

$\lambda = 5$ cm

Required: E & H at $r = 10$ cm, $\theta = 45°$

Soln: assume far field:

$\because \frac{L}{\lambda} = 3/5 = 0.6 \Rightarrow \therefore$ it is a finite length dipole

$$\frac{kL}{2} = \frac{\frac{2\pi}{\lambda} \cdot 3}{2} = \frac{\frac{2\pi}{5} \cdot 3}{2} = 0.6\pi$$

$$E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r} \left[\frac{\cos(\frac{kL}{2}\cos\theta) - \cos(\frac{kL}{2})}{\sin\theta} \right]$$

$$= j\eta \frac{I_o e^{-jkr}}{2\pi r} \left[\frac{\cos(0.6\pi \cdot \cos 45) + 0.309}{\sin 45} \right]$$

0.7703

$$E_\theta = j\,120\pi \frac{I_o e^{j60} e^{-j4\pi}}{2\pi (0.1)} (0.7703) = 4620 e^{j11.52}$$

$$H_\phi = \frac{E_\theta}{\eta}$$

$|E_\theta| = 4620$ V/m , $H_\phi = \frac{4620}{120\pi} = 12.25$ Ampere